Detailed Simulation of the Cochlea: Recent Progress Using Large Shared Memory Parallel Computers

E. Givelberg
Department of Mathematics,
University of Michigan,
Ann Arbor, MI
givelbrg@cims.nyu.edu

J. Bunn
Center for Advanced
Computing Research,
Caltech, Pasadena, CA
julian@cacr.caltech.edu

M. Rajan
Hewlett Packard Corporation,
Anaheim, CA
mrajan@cacr.caltech.edu

We have developed and are refining a detailed three-dimensional computational model of the human cochlea. The model uses the immersed boundary method to calculate the fluid-structure interactions produced in response to incoming sound waves. An accurate cochlear geometry obtained from physical measurements is incorporated. The model includes a detailed and realistic description of the various elastic structures present. Initially, a macro-mechanical computational model was developed for execution on a CRAY T90 at the San Diego Supercomputing Center. This code was ported to the latest generation of shared memory high performance servers from Hewlett Packard. Using compiler generated threads and OpenMP directives, we have achieved a high degree of parallelism in the executable, which has made possible to run several large scale numerical simulation experiments to study the interesting features of the cochlear system. In this paper, we outline the methods, algorithms and software tools that were used to implement and fine tune the code, and discuss some of the simulation results.

Background

The cochlea is the part of the inner ear where acoustic signals are transformed into neural pulses that are then signaled to the brain. It is a small snail-shell-like cavity in the temporal bone, which has two openings, the oval window and the round window. The cavity is filled with fluid and is sealed by two elastic membranes that cover the windows. The spiral canal of the cochlea is divided lengthwise into two passages by the so-called basilar membrane. These passages are the scala vestibuli and scala tympani and they connect with each other at the apex, called the helicotrema. External sounds set the ear drum in motion, which is conveyed to the inner ear by the ossicles, three small bones of the middle ear, the malleus, incus and stapes. The ossicles function as an impedance matching device, focusing the energy of the ear drum on the oval window of the cochlea. The piston-like motion of the stapes against the oval window displaces the fluid of the cochlea, so generating traveling waves that propagate along the basilar membrane.

Practically everything we know about the waves in the cochlea was discovered in the 1940s by Georg von Békésy[1] who carried out experiments in cochleae extracted from human cadavers. Despite its name the basilar membrane is in fact an elastic shell whose stiffness varies exponentially with length. (Unlike an elastic membrane, an elastic shell is not under inner tension, i.e. when it is cut the edges do not pull apart.) The basilar membrane's elastic properties have an important role in the wave mechanics of the cochlea. It is observed that a pure tone input generates a traveling wave that reaches its peak at a specific location along the basilar membrane: the position of the peak depends on the frequency of the tone. Complex sounds composed of several pure tones evoke a basilar membrane response that is similar to a superposition of the membrane's responses to the constituent frequencies. Resting on the basilar membrane is the microscopic organ of Corti, a complicated structure containing sensory receptors called hair cells. The hair cells detect fluid motion and provide input to the afferent nerve fibers that send action potentials to the brain. Thus, a pure tone input activates only a narrow band of hair cells depending on its frequency. These characteristic frequencies are monotonically decreasing along the basilar membrane from the stapes to the helicotrema.

Mathematically, the macro-mechanical system of the cochlea can be described by the Navier-Stokes equations of incompressible fluid mechanics coupled with equations describing the elastic properties of the basilar membrane and the membranes of the oval and the round windows. The mathematical problem of solving this system of partial differential equations on a three-dimensional domain with intricate curved geometry is very difficult. Since the displacements of the basilar membrane are extremely small (on a nanometer scale), the system operates in a linear regime. Analysis shows that the waves in the cochlea resemble shallow water waves[2]

While the macro-mechanics of the cochlea break up the incoming sound into its frequency components, it was suggested as early as 1948 that a passive mechanical system cannot explain the extreme sensitivity and frequency selectivity of the cochlea; some kind of amplification is necessary[3].Indeed, analysis of cochlear macro-mechanics suggests that the traveling wave focusing is not sufficiently sharp, with some estimates suggesting that, at its threshold, human hearing is about a hundred times more sensitive than expected from a passive macro-mechanical filter of the cochlea.

The study of the active mechanism of the cochlea is the ultimate goal of this project. This paper describes the preliminary step towards that goal, namely the construction of a comprehensive three-dimensional computational model of the cochlear passive macro-mechanics.

Extensive research in cochlear modeling has been carried out over the years. Because of mathematical difficulty mostly simplified one or two-dimensional models that sought to incorporate some of cochlear features have been studied. Several three-dimensional models have been reported, such as Kolston's model[4], intended to simulate the micro-mechanics of the cochlear partition in the linear regime (i.e. near the threshold of hearing), and Parthasarati, Grosh and Nuttal's[5] hybrid analytical-computational model using WKB approximations and finite-element methods. The model we have constructed is the only model that incorporates the full curved three-dimensional geometry of the cochlea, uses the viscous non-linear Navier-Stokes fluid equations, uses a shell theory to model the basilar membrane and incorporates detailed elastic models of the other materials.

The Cochlea Model

The small size and the complex geometry of the cochlea make it very difficult to measure the vibrations of the basilar membrane in vivo. We have constructed a three-dimensional computational model of the cochlea using the immersed boundary method in order to study cochlear mechanics.

The immersed boundary method of Peskin and McQueen[6] is a general numerical method for modeling an elastic material immersed in a viscous incompressible fluid. It has proved to be particularly useful for computer simulation of various biofluid dynamic systems. In this framework the elastic (and possibly active) biological tissue is treated as a collection of elastic fibers immersed in a viscous incompressible fluid. For details of this formulation of the method together with references to many applications see [6].

The immersed boundary framework is suitable for modeling not only elastic fibers, but also different elastic materials having complicated structure. The cochlea model incorporates a shell theory into the immersed boundary framework, obtaining a practical computational method for modeling an elastic shell immersed in fluid[7]. The main advantage of the method is its conceptual simplicity: the viscous incompressible fluid is described by the Navier-Stokes equations, the geometry of the model mirrors the real-life curved three-dimensional cochlear geometry and models for elastic and active material components can be naturally integrated.

The geometric model of the cochlear anatomy is based on measurements that include the position of the basilar membrane, its width and the cross-sectional area of the scalae. There are six surfaces in this model: the basilar membrane, the spiral bony shelf, the tubular walls of scala vestibuli and scala tympani and the membranes covering the oval window and the round window. The basilar membrane is modeled as an elastic shell and the oval window and the round window membranes are modeled by computational grids, whose points are interconnected by elastic springs. Figure 1 shows a partially transparent cochlea model as it is seen from the outside.

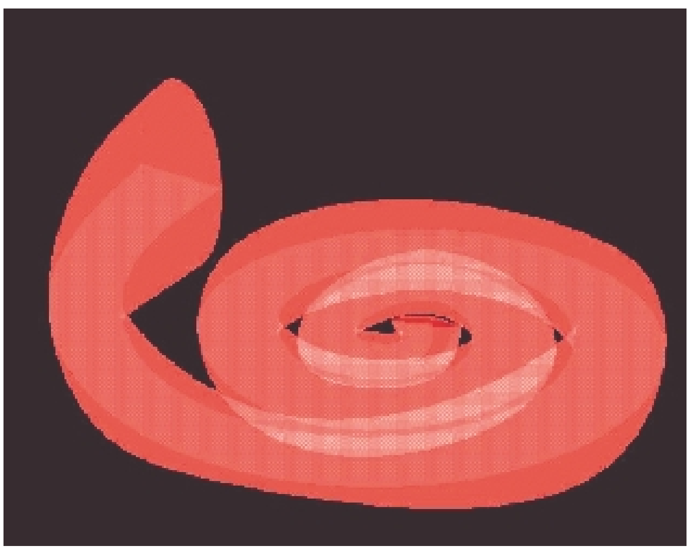

Figure 1 Partially transparent cochlea model as seen from the outside.

In Figure 2 the two external walls forming the outer tube have been removed to expose the two membranes representing the oval and the round windows and the cochlear partition consisting of a bony shelf and a narrow basilar membrane. This is a snapshot of the system taken after several time steps of the simulation algorithm had been executed. One can see the input force applied to the oval window and the resulting traveling wave beginning to propagate along the basilar membrane.

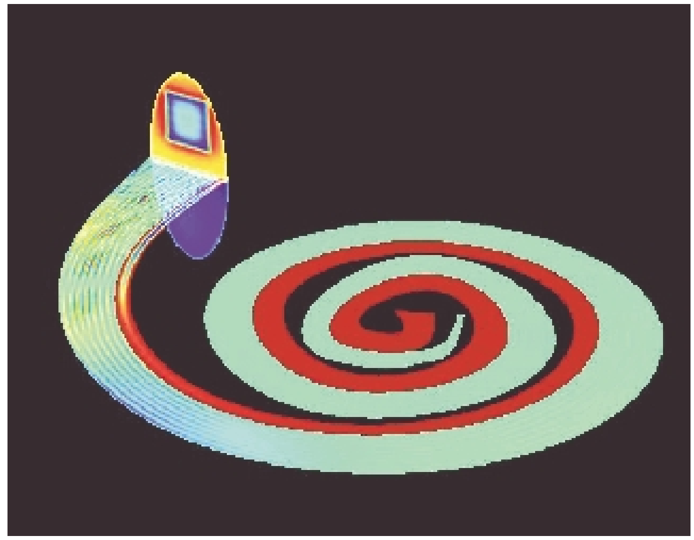

Figure 2 Snapshot of the internal parts of the system taken after several time steps of the simulation algorithm.

We use a first-order immersed boundary numerical scheme, which is the easiest to implement and has the important advantage of modularity: integration of various models of immersed elastic material is straightforward. The bulk of the computation is related to the fluid equations and to the fluid-material interaction. The solution of the fluid equations uses the Fast Fourier Transform algorithm. (see [7]).

Software Implementation

We have developed a suite of software codes to support our studies of the cochlea using the immersed boundary method. These include codes for the generation of simulation input models (implemented in C++), the main immersed boundary numerical solver, and those for the analysis and visualization of results.

The main workhorse in this suite is the general purpose immersed boundary code, which is written in C and requires extensive computing resources, in terms of CPU power, memory, and disk space for storage of the results. There are several C++ programs used to generate the input models for the immersed boundary code. The principal program generates an approximate cochlear geometry (as seen in Figures 1 and 2) and the material properties of various surfaces. To achieve approximately uniform mesh density the six surfaces of immersed material in the cochlea model are partitioned into 25 computational grids comprising approximately 750,000 points in total. They include an elastic shell modeling the basilar membrane, two elastic membranes modeling the oval and the round windows and 22 grids describing the bony walls and the shelf.

The immersed boundary code has been optimized to run on several platforms: the Silicon Graphics Origin 2000 parallel architecture, the Cray T90 vector-parallel machine and the HP 9000 V-class and Superdome systems. There is also a version for a PC running under Windows, suitable for testing small models. The code is very well instrumented with calls to system timing routines. This information proves invaluable during porting and tuning of the software on different architectures. The largest and most recent numerical experiments have been carried out on the 32 processor HP Superdome installed at the Center for Advanced Computing Research (CACR) at Caltech.

The codes that are used to analyze and visualize the simulated data include a C++/OpenGL tool that runs on SGI and a Java/Java3D tool that runs on most platforms. These tools take as input the vertex coordinates for all computational grids in the cochlea model from the result files for each time step in the simulation. The tools generate a full 3D rendering of the model geometry. Since the displacements of the basilar membrane are very small (measured by a fraction of a nano-meter at the hearing threshold), they are color-coded to reveal the dynamics of the system. An essential insight into the basilar membrane dynamics is provided by the plot of the normal displacement of its center line (see Figure 3). Other Java tools display various important characteristics of the system dynamics, such as the response of individual points on the basilar membrane as a function of time. All of the graphics tools have built-in facilities for generating animation.

The Computational Challenge

The principal practical difficulty in the implementation of the macro-mechanical model of the cochlea is due to the fact that the basilar membrane is very narrow relative to the size of the whole cochlea. In humans it is approximately 3.5 cm long and its width grows from about 150μm near the stapes to approximately 560 μm near the helicotrema, while the size of the whole cochlea is on the order of 1cm x 1cm x 1cm. Consequently, a fluid grid of at least 256^3 points is necessary to adequately resolve the small scale. In order to reduce the computing time several small changes were made to the geometric model of the cochlear anatomy. In the altered model the cochlea is packed more tightly and the whole structure fits in a half-cube, making it possible to use a fluid grid of 256 x 256 x 128 points. This configuration requires approximately 1.5 Gigabytes of main memory. Each time step of the immersed boundary computation consists of computing the forces the immersed material applied to the fluid, spreading these forces from the computational grid of the material to the computational grid of the fluid, solving the discretized Navier-Stokes equations and advancing the position of the immersed material using the newly obtained fluid velocity. The computation of the material forces is relatively inexpensive in time. The fluid solver relies primarily on the efficient parallel Fast Fourier Transform routines. The other two demanding parts of the time step required development of a specialized algorithm carefully partitioning the fluid and the material grids into portions distributed to different processors in such a way as to prevent different processors from operating simultaneously on the same portion of the data.

Extensive optimization of the immersed boundary numerical solver was necessary to make numerical experiments with the cochlea model practical. We have achieved a “wall-clock” performance of approximately 1.3 seconds per time step for the cochlea model on the HP Superdome running in a dedicated mode. The HP 9000 Superdome at CACR contains 32 RISC processors arranged in a cell-based hierarchical crossbar architecture, with each cell consisting of 4 cpus with 8Gb of memory and an I/O sub-system. This architecture supports the shared memory programming model and the code efficiency was achieved primarily through the use of OpenMP parallelization directives.

In addition to the code instrumentation several software tools such as the HP CXperf and the KAI Guideview were used to display parallel efficiency of every section in the code and to examine data cache and TLB misses. As a result, the cochlea code achieves excellent scaling on the Superdome, as depicted in Figure 3.

Figure 3: Wall clock time in seconds per one time-step vs. the number of system processors.

Simulation Results

The achieved efficiency of the immersed boundary solver allowed several large numerical experiments to be successfully completed. Each experiment consisted of providing an input of a pure tone of a given frequency at the stapes and the study of the subsequent motion of the basilar membrane. A very small time step of 40 nano-seconds was chosen to guarantee numerical stability. A typical experiment was run for 50,000 time steps, equivalent to 2 milliseconds of simulated time, and took approximately 18 hours of dedicated computation on the Superdome.

There have been many experimental and modeling studies of cochlear response to a pure tone input and our simulations reproduce the characteristic features of the cochlear macro-mechanics. We have run experiments with varying input sound frequencies of 20, 10, 5 and 2 KHz. In each instance we observe a traveling wave propagating from the stapes in the direction of the helicotrema. The amplitude of the wave is gradually increasing until it reaches a peak at a characteristic location along the basilar membrane depending on the input frequency. The higher the frequency the closer the peak is to the stapes. Another characteristic feature observed was that after reaching the peak the wave drops off very sharply, essentially shutting down. Figure 4 shows a snapshot of a single time step of a 10Khz experiment, as displayed by the Java analysis tool. The wave pattern on the basilar membrane (black line) can be clearly seen. The red lines indicate the envelope of the wave computed over a series of previous time steps.

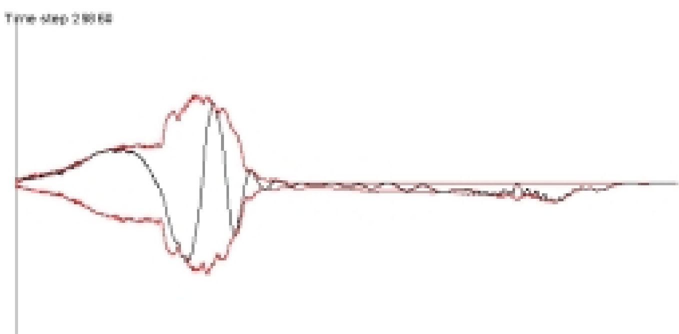

Figure 4 Normal displacement of the center line of the basilar membrane after 23850 time steps. The peak displacement is approximately 0.5 nanometers.

The interested reader is invited to view several animations of our results by visiting the Web site:

http://pcbunn.cacr.caltech.edu/Cochlea/default.html

Conclusion

We have constructed a comprehensive three-dimensional computational model of the cochlea using the immersed boundary method. Extensive optimization and parallelization made it possible to complete several large-scale numerical simulations on a 32-processor shared memory HP Superdome computer. The pure tone experiments capture the most important properties of the cochlear macro-mechanics. We intend to continue testing the model and compare the results with the available experimental and modeling data. For example, we would like to construct a cochlear map for this model, i.e. to determine the characteristic frequency of each location on the basilar membrane. Subsequently we will extend the present model to capture the micro-mechanics of the organ of Corti in order to study the microscopic mechanism of amplification in the cochlea. We anticipate such a model to be even more computationally demanding than the present model.

We would like to thank Sarah Emery Bunn for her assistance in the preparation of the manuscript.

[1] G. von Békésy, *Experiments in Hearing* (E.G. Weaver trans.), McGraw Hill, New York, 1960

[2] Randall J. Leveque, Charles S. Peskin, Peter D. Lax, Solution of a Two-dimensional Cochlea Model with Fluid Viscosity. *SIAM J. Appl. Math*., 48, No.1, (1988), 191-213.

[3] Gold, T. Hearing II. The physical basis of the action of the cochlea. *Proc R. Soc Lond [Biol]* 1948, 135:492-498

[4] Kolston, P.J., Finite element micromechanical modeling of the cochlea in three dimensions. *J. Acoust. Soc. Am.* Vol.99, (1) 455-467

[5] Parthasarati, A. A., Grosh, Kl, Nuttall, A.L. Three-dimensional numerical modeling for global cochlear dynamics. *J. Acoust. Soc. Am.*, 107(1), Jan 2000, 474-485

[6] Charles S. Peskin, David M. McQueen, A General Method for the Computer Simulation of Biological Systems Interacting with Fluids. *Proc. Of SEB Symposium on Biological Fluid Dynamics, Leeds, England*, July 5-8, 1994

[7] E. Givelberg, *Modeling Elastic Shells Immersed in Fluid*. Ph.D. Thesis, NYU, 1997